Journal of Engineering Physics and Thermophysics, Vol. 67, Nos. 5-6, 1994

INVESTIGATION OF HEAT TRANSFER IN A PLANE CHANNEL WITH DISCRETE HEAT SUPPLY WITH ACCOUNT FOR THE AXIAL THERMAL CONDUCTIVITY OF THE WALL

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We present an analytical solution for the problem of convective heat transfer and the results of calculations of heat transfer in channels with discrete heat release in laminar and slug flows.

The problem of the study of convective heat transfer in channels with discrete heat supply to the walls arises in creating systems for achieving the normal thermal regime of radioelectronic equipment and, in particular, modern antenna systems.

In investigations carried out to date, heat transfer from microcircuits in channels formed by printed circuit boards with the microcircuits was usually determined [1, 2]. Analytical, mainly numerical, and experimental investigations on heat transfer in plane channels with discrete sources on the walls are presented in [3–5].

In [3] results of a numerical investigation of heat transfer in channels with discrete heat release in the walls are given in a conjugate formulation on the initial hydrodynamic and thermal section; however, no computational relations are presented.

In [4] results of a numerical solution of a conjugate problem with a developed velocity profile are compared with results of experimental investigations of four rows of heat generating modules located on one of the channel walls. Experimental data that exceed somewhat the predicted results on heat transfer in the region of laminar flow as well as good coincidence of the results of calculation with the experiment for turbulent flow are noted. Investigations of heat transfer from discrete modules of constant temperature in a turbulent flow are presented in [5]. In that work a numerical solution of the corresponding problem in a nonconjugate formulation is also presented.

In the above-listed works a number of characteristic trends are noted that are typical of the case of heat transfer considered. Thus, sometimes with discrete heat supply the heat transfer coefficients in its sections may exceed the heat transfer coefficients at that place in the channel for the case of constant heat supply by more than a factor of two. Despite the above, as noted in [5], where separation of the heat supply sections by heat-insulating gaps is considered from the standpoint of heat transfer enhancement, the total amount of heat removed from a channel of fixed length does not increase.

However, up to the present time the problem of heat transfer in channels with discrete heat supply in a conjugate formulation has not been solved analytically. Heat transfer from heat generation sources at a considerable distance from the inlet to the channel has not received sufficient study although knowledge of it is necessary for determining the parameters of the systems for cooling antenna arrays with a large number of parallel long channels as well as the effect of the thermal conductivity and thickness of the wall on heat transfer.

In the present work we present accurate analytical solutions of the problems of convective heat transfer with account for the axial thermal conductivity of the walls for laminar and slug flows of the heat carrier and discrete change in the heat flux density on the walls and we investigate the main laws governing convective heat transfer that characterize this case. We pose the problem as follows (see Fig. 1).

We write down the energy equation for the liquid flow and the wall:

Institute of Precision Mechanics and Optics, St. Petersburg, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 67, Nos. 5-6, pp. 405-417, November-December, 1994. Original article submitted August 12, 1992; revision submitted April 2, 1994.

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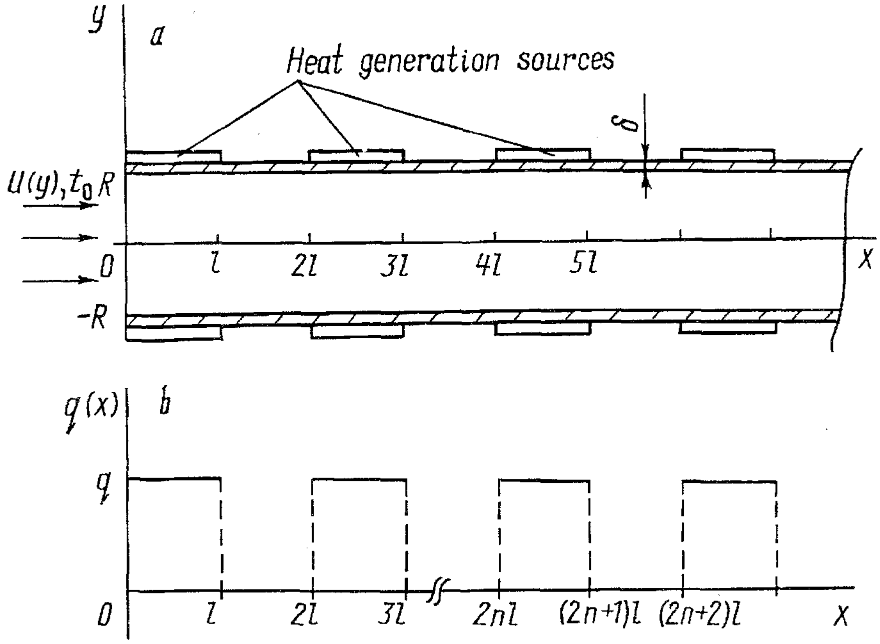

Fig. 1. A plane channel with discrete heat generation sources on the walls (a) and the change in the heat flux density on the walls as a function of x (b).

$$U(y)\frac{\partial t_2}{\partial x} - a\frac{\partial^2 t_2}{\partial y^2} = 0\,, \tag{1}$$

$$\frac{d^2 t_1}{dx^2} - \frac{\lambda_2}{\lambda_1 \delta} F(x) = -\frac{q(x)}{\lambda_1 \delta}\,, \tag{2}$$

where $U = U_0$ for the slug velocity profile; $U(y) = U_{\mathrm{m}}(1 - (y/R)^2)$ for the laminar velocity profile, $U_m = 3/2\overline{U}$.

The boundary and conjugation conditions have the form

$$t_2|_{x=0} = t_0\,, \quad t_2|_{y=R} = t_1(x)\,,$$

$$\left.\frac{\partial t_2}{\partial y}\right|_{y=0} = 0\,, \quad \left.\frac{\partial t_2}{\partial y}\right|_{y=R} = F(x)\,. \tag{3}$$

In writing down Eq. (2), in formulating the problem, we made the following assumptions:

the temperature difference over the wall thickness is not taken into account (it is assumed to be small);

the outer surfaces of the channel walls are assumed to be adiabatic.

First, we present a solution for a developed laminar liquid flow.

We consider symmetric discrete heat supply with equal powers of each of the sources and width of a source l equal to the spacing between the sources (see Fig. 1). As is known, in the formulation considered, Eq. (2) of the system may be treated as a complex boundary condition for $t_2(x, y)$. The essence of the proposed method for solving the problem formulated is as follows. The discrete, periodically varying (with the period $2l$) heat flux density $q(x)$ on the walls is represented as a Fourier series on the segment $2nl < x < (2n + 2)l$, $n = 0, 1, 2, \ldots$:

$$q(x) = \frac{q}{2} + \sum_{k=1}^{\infty} \frac{q}{k\pi}\,[1 - (-1)^k]\sin\frac{k\pi}{l}\,x\,. \tag{4}$$

Passing to the variables x and $\xi = y/R$ and denoting $\vartheta = t - t_0$, we seek the solution of Eq. (1) in the form of the sum $\vartheta = \vartheta_{\mathrm{en}} + \vartheta_*$, where ϑ_* is the solution that determines the temperature field at a distance from the inlet to the channel ($x \to \infty$) and ϑ_{en} is the solution determining the temperature field in the entry section. The equation and boundary conditions for ϑ_* in the variables x, ξ have the form

$$\frac{U_m R^2}{a}(1-\xi^2)\frac{\partial \vartheta_*}{\partial x} - \frac{\partial^2 \vartheta_*}{\partial \xi^2} = 0, \tag{5}$$

$$\left.\frac{\partial \vartheta_*}{\partial \xi}\right|_{\xi=0} = 0; \quad \frac{d^2 \vartheta_*|_{\xi=1}}{dx^2} - \frac{\lambda_2}{\lambda_1 \delta R}\left.\frac{\partial \vartheta_*}{\partial \xi}\right|_{\xi=1} = -\frac{q(x)}{\lambda_1 \delta}. \tag{6}$$

Because of the periodicity of the heat supply law, we suggest seeking the solution $\vartheta_*(x, \xi)$ in the form of the sum of a term linearly increasing with increase in x and a certain periodic function with the period of variation over x equal to the period of variation of the heat flux density. In this case the periodic function is represented as a Fourier series on a segment of length $2l$, equal to the period of variation of $q(x)$, with coefficients that are certain functions of ξ, i.e.,

$$\vartheta_*(x, \xi) = Ax + \frac{a_0(\xi)}{2} + \sum_{k=1}^{\infty} a_k(\xi)\cos\frac{k\pi}{l}x + b_k(\xi)\sin\frac{k\pi}{l}x. \tag{7}$$

Substituting ϑ_* into Eq. (5) to determine the functions $a_k(\xi)$ and $b_k(\xi)$, we obtain a system of ordinary differential equations of second order:

$$b_k''(\xi) + D(1-\xi^2)a_k(\xi) = 0, \quad a_k''(\xi) - D(1-\xi^2)b_k(\xi) = 0, \tag{8}$$

where

$$D = \frac{k\pi R^2 U_m}{la},$$

and an equation for $a_0(\xi)$:

$$\frac{a}{2R^2}a_0''(\xi) = AU_m(1-\xi^2). \tag{9}$$

The solution of system (8) is determined in the form of the power series

$$a_k(\xi) = \sum_{n=0}^{\infty} e_{nk}\xi^n, \quad b_k(\xi) = \sum_{n=0}^{\infty} g_{nk}\xi^n. \tag{10}$$

Substituting these series into system (8), we can easily obtain recurrence relations to determine the coefficients e_{nk} and g_{nk}. These relations have the form

$$De_{0k} + 2g_{2k} = 0, \quad De_{1k} + 6g_{3k} = 0,$$

$$Dg_{0k} - 2e_{2k} = 0, \quad Dg_{1k} - 6e_{3k} = 0, \tag{11}$$

Satisfying boundary conditions (6), we obtain that the coefficients e_{nk} and g_{nk} are equal to zero for odd values of n, and we obtain following system of equations (for each k) to determine e_{0k} and g_{0k}:

$$D(e_{nk} - e_{n-2k}) + g_{n+2k}(n+2)(n+1) = 0, \quad n \ge 2,$$

$$D(g_{nk} - g_{n-2k}) - e_{n+2k}(n+2)(n+1) = 0, \quad n \ge 2,$$

$$k = 1, 2, 3, \ldots$$

$$e_{0k}\left(\frac{k\pi}{l}\right)^2 + \sum_{n=2}^{\infty}\left[\left(\frac{k\pi}{l}\right)^2 + \frac{\lambda_2}{\lambda_1\delta R}n\right]e_{nk} = 0,$$

$$g_{0k}\left(\frac{k\pi}{l}\right)^2 + \sum_{n=2}^{\infty}\left[\left(\frac{k\pi}{l}\right)^2 + \frac{\lambda_2}{\lambda_1\delta R}n\right]g_{nk} = \frac{q}{\lambda_1\delta k\pi}\,[1 - (-1)^k], \tag{12}$$

$n = 2, 4, 6 \ldots$, in the sums.

The algebraic system considered can be solved at each k by representing the coefficients e_{nk} and g_{nk} in terms of e_{0k} and g_{0k} in the following way:

$$g_{nk} = C_{nk}\,g_{0k} + D_{nk}\,e_{0k}, \quad n \geq 2, \quad e_{nk} = B_{nk}\,g_{0k} + F_{nk}\,e_{0k}.$$

In this case the coefficients C_{nk}, D_{nk}, B_{nk}, and F_{nk} introduced are determined from recurrence relations of type (11), while $C_{2k} = F_{2k} = 0$, $D_{2k} = -D/2$, $B_{2k} = D/2$ have already been determined. For $k = 2, 4, 6, \ldots$ in the case considered the solution of (12) is trivial, e_{0k} and $g_{0k} = 0$. The coefficient $a_0(\xi)$ in Eq. (7) and the constant A are determined by direct integration of Eq. (9). Substituting the expressions obtained into formula (7), for the temperature field at a distance from the inlet to the channel we finally obtain

$$\vartheta_*(x, \xi) = \frac{3}{4}\frac{aq}{U_m R\lambda_2}x + \frac{3}{8}\frac{qR}{\lambda_2}\xi^2 - \frac{1}{16}\frac{qR}{\lambda_2}\xi^4 + \sum_{k=1}^{\infty} a_k(\xi)\cos\frac{k\pi}{l}x + b_k(\xi)\sin\frac{k\pi}{l}x. \tag{13}$$

We now find the solution $\vartheta_{en}(x, \xi)$. The equations and boundary conditions for determining ϑ_{en} are similar to Eqs. (5) and (6), but the second boundary condition in Eqs. (6) is homogeneous. The solution is sought by the method of separation of variables. Substituting $\vartheta_{en}(x, \xi)$ in the form of the sum of products $\vartheta_{en} = \sum_i^{\infty} X_i(x)Y_i(\xi)$, for $X_i(x)$ we immediately obtain $X_i = \exp(-(\gamma_i/s)\cdot x)$, where $s = U_m R^2/a$; γ_i are the eigenvalues. To determine $Y_i(\xi)$, we have the following boundary-value problem:

$$Y'' + \gamma(1 - \xi^2)\,Y = 0, \tag{14}$$

$$Y'|_{\xi=0} = 0; \quad Y|_{\xi=1} - \frac{\lambda_2}{\lambda_1\delta R}\left(\frac{s}{\gamma_i}\right)^2 Y'|_{\xi=1} = 0, \tag{15}$$

The quantity $Y_i(\xi)$ is sought in the form of a series, just as for the solution of a nonconjugate problem [6]:

$$Y_i = \sum_{n=0}^{\infty} A_n\xi^n. \tag{16}$$

Substituting this expression into Eq. (14), we obtain a recurrence relation for A_n:

$$A_{n+2} = \gamma_i\frac{A_{n-2} - A_n}{(n+1)(n+2)}, \quad A_0 = 1; \quad A_2 = -\frac{\gamma_i}{2}. \tag{17}$$

From the first boundary condition of Eqs. (15) it follows that all the coefficients A_n for odd values of n are equal to zero (i.e., $n = 0, 2, 4, \ldots$). The second boundary condition of Eqs. (15) yields an equation for determining the eigenvalues γ_i:

$$A_0 + \sum_{n=2}^{\infty} A_n\left(1 - \frac{\lambda_2}{\lambda_1\delta R}\left(\frac{s}{\gamma_i}\right)^2 n\right) = 0. \tag{18}$$

To find the eigenvalues at large values of γ_i, we shall avail ourselves of the method suggested by Cess [7], who used the asymptotic solution of Eq. (14). Satisfying the second boundary condition of Eqs. (15) in the asymptotic solution and performing the necessary transformations, we write the final equation for determining the eigenvalues ($\beta_i^2 = \gamma_i$) of the formulated problem at large values of γ_i (β_i):

$$\beta^{\frac{20}{6}} = 0.45922 \frac{\lambda_2 s^2}{\lambda_1 \delta R} + 0.79539 \frac{\lambda_2 s^2}{\lambda_1 \delta R} \operatorname{ctan} \left(\frac{(3\beta - 5)}{12} \pi \right). \tag{19}$$

The eigenvalues γ_i (β_i) calculated by Eqs. (18) and (19) are presented in Tables 1 and 2. As seen from a comparison of the eigenvalues from these tables, Eq. (19) can be used for determining β_i (γ_i) when $i \geq 7$. However, already at $i \geq 5$ the dependence of the eigenvalues on the combination $\lambda_2 s^2/\lambda_1 \delta R$ vanishes, and therefore when $i \geq 7$ these values can be determined from the approximate formula (which follows from Eq. (19))

$$\beta_i = 4i - 7/3. \tag{19a}$$

When $i < 7$, the values of β_i should be determined directly by numerical solution of Eq. (18) (see Table 1).

Thus, the solution for the entry section can be written in the form

$$\vartheta_{st}(x, \xi) = \sum_{i=1}^{\infty} B_i Y_i(\xi) \exp\left(-\frac{\gamma_i}{s} x \right). \tag{20}$$

The coefficients B_i are determined from the initial condition at x = 0: $\vartheta_{st}|_{x=0} = -\vartheta_*|_{x=0}$. In this case, since the second boundary condition of Eqs. (15) (similar to the usual boundary condition of the third kind) involves the eigenvalue γ_i, the eigenfunctions $Y_i(\xi)$ are not orthogonal, and the determination of B_i is made in a somewhat more complex way. We write out the final formulas for determining B_i:

$$B_i = \frac{\int_0^1 G(\xi)\, Y_i(\xi)\, (1 - \xi^2)\, d\xi + S Y_i(1)\, (\gamma_i G(1) + W)}{\int_0^1 Y_i^2(\xi)\, (1 - \xi^2)\, d\xi + 2S\gamma_i Y_i^2(1)}. \tag{21}$$

The integral in the denominator is:

$$I_1 = \int_0^1 Y_i^2(\xi)\, (1 - \xi^2)\, d\xi = \sum_{j=0}^{\infty} \sum_{n=0}^{\infty} A_j A_n \frac{2}{(j + n + 1)(j + n + 3)},$$

and the integral in the numerator is

$$I_2 = \int_0^1 G(\xi)\, Y_i(\xi)\, (1 - \xi^2)\, d\xi = \frac{qR}{\lambda_2} \sum_{n=0}^{\infty} A_n \left(\frac{7}{16(n + 5)} - \frac{3}{8(n + 3)} - \frac{1}{16(n + 7)} \right) +$$

$$+ \sum_{k=1}^{\infty} \sum_{j=0}^{\infty} \sum_{n=0}^{\infty} A_j e_{nk} \frac{-2}{(n + j + 3)(n + j + 1)}.$$

The constants S, W, and $G(1)$ are

$$S = \frac{\lambda_1 \delta R}{\lambda_2 s}, \quad W = \frac{G(1) - \sum_{n=2}^{\infty} Y_n(1) \frac{I_2 + S Y_n(1)\, \gamma_n G(1)}{I_1 + 2S\gamma_n Y_n^2(1)}}{\sum_{n=2}^{\infty} \frac{S Y_n^2(1)}{I_1 + 2S\gamma_n Y_n^2(1)}},$$

TABLE 1. Eigenvalues of the Problem (the first six or seven values)

i	β_i	$\nu_i(\beta_i^2)$	i	β_i	$\nu_i(\beta_i^2)$	i	β_i	$\nu_i(\beta_i^2)$
	$L^{-1}=1/L=\frac{\lambda_2 s^2}{\lambda_1 \delta R}=17,40$			$L^{-1}=390,6$			$L^{-1}=3125$	
1	2,5752	6,6318	1	3,9394	15,519	1	4,2368	17,950
2	5,7237	32,761	2	6,5714	43,183	2	7,8648	61,855
3	9,6775	93,653	3	9,8753	97,523	3	10,958	120,08
4	13,671	186,88	4	13,733	188,60	4	14,204	201,74
5	17,669	312,18	5	17,695	313,12	5	17,898	320,32
6	21,674	460,76	6	21,683	470,14	6	21,782	474,44
7	25,371	643,69	7	25,258	637,97	7	25,448	647,59
	$L^{-1}=78,13$			$L^{-1}=1392$			$L^{-1}=3479$	
1	3,2777	10,743	1	4,1766	17,444	1	4,2418	17,993
2	5,9018	34,831	2	7,4394	55,345	2	7,9061	62,507
3	9,7095	94,274	3	10,365	107,44	3	11,045	121,99
4	13,681	187,16	4	13,906	193,38	4	14,262	203,40
5	17,673	312,33	5	17,768	315,69	5	17,925	321,29
6	21,668	469,51	6	21,715	471,56	6	21,794	474,98
7	25,348	642,50	7	25,348	642,50	7	25,357	642,99
	$L^{-1}=174,0$			$L^{-1}=19,53$			$L^{-1}=3906$	
1	3,6493	13,317	1	4,2074	17,702	1	4,2467	18,034
2	6,1486	37,806	2	7,6413	58,390	2	7,9470	63,155
3	9,7604	95,265	3	10,592	112,19	3	11,139	124,08
4	13,697	187,60	4	14,004	196,10	4	14,331	205,37
5	17,680	312,57	5	17,809	317,17	5	17,957	322,46
6	21,678	469,94	6	21,739	472,57	6	21,810	475,67
7	25,357	642,96	7	25,372	643,75	7	25,685	659,71
	$L^{-1}=6959$			$L^{-1}=15625$			$L^{-1}=39062$	
1	4,2643	18,185	1	4,2769	18,292	1	4,2831	18,345
2	8,0994	65,601	2	8,2121	67,439	2	8,2671	68,345
3	11,566	133,78	3	11,965	143,15	3	12,173	148,18
4	14,758	217,79	4	15,440	238,39	4	15,953	254,50
5	18,190	330,87	5	18,768	352,22	5	19,554	382,34
6	21,932	481,00	6	22,260	495,50	6	23,021	529,97
7	25,613	655,99	7	25,856	669,25	7	—	—
	$L^{-1}=7813$			$L^{-1}=17397$			$L^{-1}=195313$	
1	4,2669	18,206	1	4,2781	18,302	1	4,2865	18,374
2	8,1213	65,956	2	8,2214	67,591	2	8,2965	68,831
3	11,639	13,547	3	11,999	143,99	3	12,284	150,88
4	14,856	220,71	4	15,519	240,85	4	16,245	263,89
5	18,253	333,19	5	18,864	355,85	5	20,167	406,71
6	21,960	482,24	6	22,305	497,50	6	23,707	561,99
7	25,688	659,87	7	25,593	654,99	7	—	—
	$L^{-1}=13918$			$L^{-1}=31250$				
1	4,2758	18,282	1	4,2822	18,337			
2	8,2009	67,255	2	8,2579	68,192			
3	11,923	142,15	3	12,138	147,34			
4	15,350	235,61	4	15,862	251,59			
5	18,668	348,50	5	19,383	375,70			
6	22,200	492,84	6	22,776	518,73			
7	25,612	655,97	7	25,476	648,99			

Remark. When $i \geq 7$, the eigenvalues can be calculated from the approximating relation (19a) or by solving Eq. (19) for large values of β. Eigenvalues found from the approximate equation are presented in Table 2.

TABLE 2. Eigenvalues β_i ($\gamma_i = \beta_i^2$) Determined by Solving Eq. (19)

L^{-1}	i								
	1	2	3	4	5	6	7	8	9
0.303	1.6750	5.6672	9.6672	13.667	17.667	21.667	25.667	29.667	33.667
30.25	1.9660	5.6672	9.6672	13.667	17.667	21.667	25.667	29.667	33.667
151.2	2.2638	5.6672	9.6672	13.667	17.667	21.667	25.667	29.667	33.667
$1.9 \cdot 10^2$	2.3195	5.66681	9.6672	13.667	17.667	21.667	25.667	29.667	33.667
$3.0 \cdot 10^2$	2.4240	5.6681	9.6672	13,.667	17.667	21.667	25.667	29.667	33.667
$7.6 \cdot 10^2$	2.6623	5.6711	9.6672	13.667	17.667	21.667	25.667	29.667	33.667
$1.5 \cdot 10^3$	2.8615	5.6750	9.6672	13.667	17.667	21.667	25.667	29.667	33.667
$3.0 \cdot 10^3$	3.0734	5.6828	9.6672	13.667	17.667	21.667	25.667	29.667	33.667
$2.4 \cdot 10^4$	3.6662	5.7843	9.6701	13.667	17.667	21.667	25.667	29.667	33.667
$3.0 \cdot 10^4$	3.6662	5.8107	9.6701	13.667	17.667	21.667	25.667	29.667	33.667
$7{,}6 \cdot 10^4$	3.6628	5.9831	9.6784	13.671	17.671	21.671	25.671	29.671	33.671
$3.5 \cdot 10^5$	3.6662	5.5343	9.7111	13.671	17.667	21.667	25.667	29.667	33.667
$3.5 \cdot 10^6$	3.662	7.6662	10.060	13.706	17.673	21.668	25.667	29.667	33.667
$3.5 \cdot 10^7$	3.662	7.6662	11.328	14.045	17.734	21.683	25.671	29.668	33.667

Remark. The values of the first eigenvalues (for different values of $L^{-1} = 1/L = (\lambda_2 U_m^2 R^3)/(\lambda_1 \delta a^2)$) listed in the table should not be used in calculations; the data of Table 1 are used for that purpose. They are given only to illustrate the trend of the eigenvalues toward values that are independent of L^{-1} at large values of i ($i \geq 5$). In this case the disappearance of the dependence on L^{-1} is observed earlier than the coincidence of the eigenvalues with those calculated by the accurate equation (see Table 1) at $i = 7$, and therefore to determine eigenvalues at $i \geq 7$, relation (19a) may be used.

$$G(1) = -\frac{5}{16}\frac{qR}{\lambda_2} - \sum_{k=1}^{\infty} a_k(1).$$

Summation in the sums presented above is made over even values of n and j, $n = 0, 2, 4, \ldots, j = 0, 2, 4, \ldots$.

Thus, the unknown values entering into formula (20) and needed for determining the solution ϑ_{en} in the entry section have all been determined.

Joining solutions (20) and (13) respectively for the entry section and the section of the channel at a distance from the inlet, we obtain the final solution for the temperature field $\vartheta(x, \xi)$ over the entire length of the channel with a laminar flow:

$$\vartheta(x, \xi) = \frac{3}{4}\frac{aq}{U_m R \lambda_2} x + \frac{3}{8}\frac{qR}{\lambda_2}\xi^2 - \frac{1}{16}\frac{qR}{\lambda_2}\xi^4 +$$

$$+ \sum_{k=1}^{\infty} a_k(\xi)\cos\frac{k\pi}{l}x + b_k(\xi)\sin\frac{k\pi}{l}x + \sum_{i=1}^{\infty} B_i \exp\left(-\frac{\gamma_i}{s}x\right) Y_i(\xi). \tag{22}$$

The channel wall temperature $t_1(x)$ is determined from Eq. (22) at $\xi = 1$, i.e., $t_1 = t_0 + \vartheta(x, 1)$, $\vartheta_{\text{w}} = \vartheta(x, 1)$, and the local number Nu_x is found from the formula

$$\text{Nu}_x = \frac{\alpha_x h}{\lambda_2} = \frac{2\left.\frac{\partial\vartheta}{\partial\xi}\right|_{\xi=1}}{\vartheta_{\text{w}} - \bar{\vartheta}}, \tag{23}$$

where ϑ_w is the wall temperature; $\bar{\vartheta}$ is the mass-mean temperature in the section considered.

Omitting intermediate calculations, we write down an expression for the local number Nu_x:

$$Nu_x = \left\{ 1 + 2\frac{\lambda_2}{qR} \sum_{k=1}^{\infty} \left[\left(\sum_{n=2}^{\infty} e_{nk} n \right) \cos\frac{k\pi}{l} x + \left(\sum_{n=2}^{\infty} g_{nk} n \right) \sin\frac{k\pi}{l} x \right] + \right.$$

$$\left. + 2\frac{\lambda_2}{qR} \sum_{i=1}^{\infty} B_i \left(\sum_{n=2}^{\infty} A_n n \right) \exp\left(-\frac{\gamma_i}{s} x \right) \right\} \times$$

$$\times \left\{ 0.24286 + \frac{\lambda_2}{qR} \sum_{k=1}^{\infty} \left[\left(\sum_{n=2}^{\infty} e_{nk} \frac{n(n+4)}{(n+1)(n+3)} \right) \cos\frac{k\pi}{l} x + \right. \right.$$

$$\left. + \left(\sum_{n=2}^{\infty} g_{nk} \frac{n(n+4)}{(n+1)(n+3)} \right) \sin\frac{k\pi}{l} x \right] +$$

$$\left. + \frac{\lambda_2}{qR} \sum_{i=1}^{\infty} B_i \left(\sum_{n=2}^{\infty} A_n \frac{n(n+4)}{(n+1)(n+3)} \right) \exp\left(\frac{\gamma_i}{s} x \right) \right\}^{-1}. \quad (24)$$

The summation is made over even values of n. Assuming that $x \to \infty$ and $\bar{e}_{nk} = (\lambda_2/qR)e_{nk}$, $\bar{g}_{nk} = (\lambda_2/qR)g_{nk}$, $\bar{B}_i = (\lambda_2/qR)B_i$ we obtain in formula (24) an expression for the local Nusselt number at a distance from the inlet of the channel; in this case $\bar{e}_{nk}$, $\bar{g}_{nk}$, and $\bar{B}_i$ no longer depend on q:

$$Nu_{x\,\mathrm{dis}} = \left\{ 1 + 2 \sum_{k=1}^{\infty} \left[\left(\sum_{n=2}^{\infty} \bar{e}_{nk} n \right) \cos\frac{k\pi}{l} x + \left(\sum_{n=2}^{\infty} \bar{g}_{nk} n \right) \sin\frac{k\pi}{l} x \right] \right\} \times$$

$$\times \left\{ 0.24286 + \sum_{k=1}^{\infty} \left[\left(\sum_{n=2}^{\infty} \bar{e}_{nk} \frac{n(n+4)}{(n+1)(n+3)} \right) \cos\frac{k\pi}{l} x + \right. \right.$$

$$\left. \left. + \left(\sum_{n=2}^{\infty} \bar{g}_{nk} \frac{n(n+4)}{(n+1)(n+3)} \right) \sin\frac{k\pi}{l} x \right] \right\}^{-1}. \quad (25)$$

As is seen from the formula obtained, the local number $Nu_{x\,\mathrm{dis}}$ for discrete heat supply has an oscillating character as a function of x, i.e., at a distance from the inlet of the channel a quasistabilized heat transfer law holds with respect to x.

The solution of the problem for slug liquid flow is achieved in a similar way.

Results of Calculations and Their Analysis. Using the formulas obtained for both the laminar and slug flow we performed calculations of the local and mean (over the section of heat supply) Nusselt numbers in a wide range of parameters (with the width of the source equal to the spacing between the sources).

The dependence of the local Nusselt number on the longitudinal coordinate for different thermal conductivities of the wall for an air flow are presented in Fig. 2 respectively for the thermal entrance region (the data are given only for slug flow) and the region at a distance from the inlet of the channel. For comparison the figure contains dependences for the case of uniform heat supply over the entire channel length. As seen from the results presented, the heat transfer coefficient at the site of the discrete source can exceed substantially the heat transfer coefficient at this site for the case of uniform heat supply. This is confirmed by data obtained earlier by other investigators [3, 5].

An analysis of the formulas obtained for the local Nusselt number shows that this criterion depends on the dimensionless combinations: $Pe = Uh/a$, $\Pi = (\lambda_1/\lambda_2)\cdot(\delta/l)$ and also on R/l ($h = 2R$), or $Nu_x = f(\Pi, Pe, R/l, x/R)$. It is clear that when the width of a source is not equal to the spacing between the sources, the indicated groups are supplemented with the ratio l/L, where L is the spacing between the sources along the heat carrier path. The

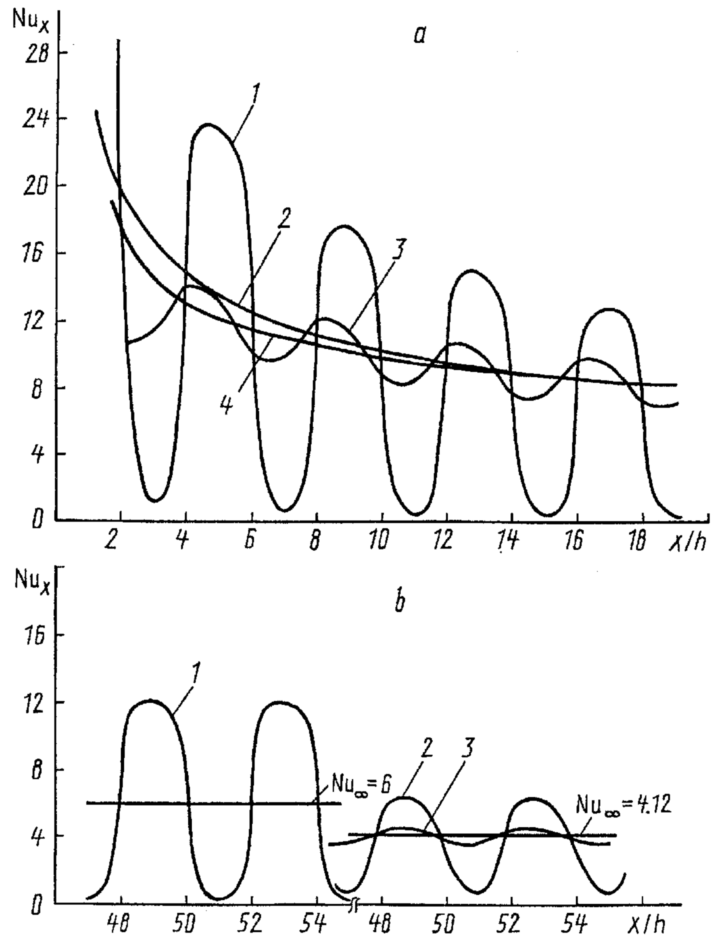

Fig. 2. Variation of the local Nusselt number ($Nu_x = \alpha_x h/\lambda_2$) with the longitudinal coordinate x/h in the case of discrete and uniform heat supply with Pe = 856; a) entrance section; discrete heat supply, $R/l = 0.25$, slug flow: 1) $\Pi^* = 0.283$, $\lambda_1 = 0.3$ W/(m·K); 3) 9.43 and 10; uniform heat supply: 2) $\lambda_1 = 0.3$ W/(m·K); 4) 10; b) at a substantial distance from the inlet of the channel; discrete heat supply, $R/l = 0.25$; 1) slug flow $\Pi^* = 0.283$; 2) laminar, $\Pi^* = 0.283$; 3) laminar, $\Pi^* = 4.717$.

number Π has a clear physical meaning: at the same temperature gradients in the liquid perpendicular to the wall near a source and in the wall along it, into the portion adjacent to the source without heat release, it represents the ratio of the power removed by convection directly from the surface of the source into the air and that removed by conduction to the portion of the wall adjacent to the source (for subsequent transfer likewise to the air in the channel).

In determining the mean Nusselt number Nu_{sec} for the section of heat generation as the integral mean one over the section of heat supply, we performed calculations of its dependence on the indicated dimensionless combinations Π and Pe. The dependence of the mean number Nu_{sec} over the section of heat supply for the sections at a distance from the inlet of the channel on the number Π, which takes into account the thermal conductivity and thickness of the channel wall, is presented in Fig. 3. As is seen from this graph, as the number Π increases (which corresponds, for example, to an increase in the thermal conductivity of the wall), the number Nu_{sec} at all the values of Pe tends to the value 4.1, which corresponds to heat transfer in the case of uniform heat supply over the entire length of the channel. From this graph it is also seen that with a decrease in Π the heat transfer coefficient increases, tending to a certain value determined by the parameter Pe at constant values of R/l and l/L, i.e., actually to the value of the Nusselt number obtained from the solution of the corresponding nonconjugate problem (without allowance for the axial thermal conductivity of the wall) [8].

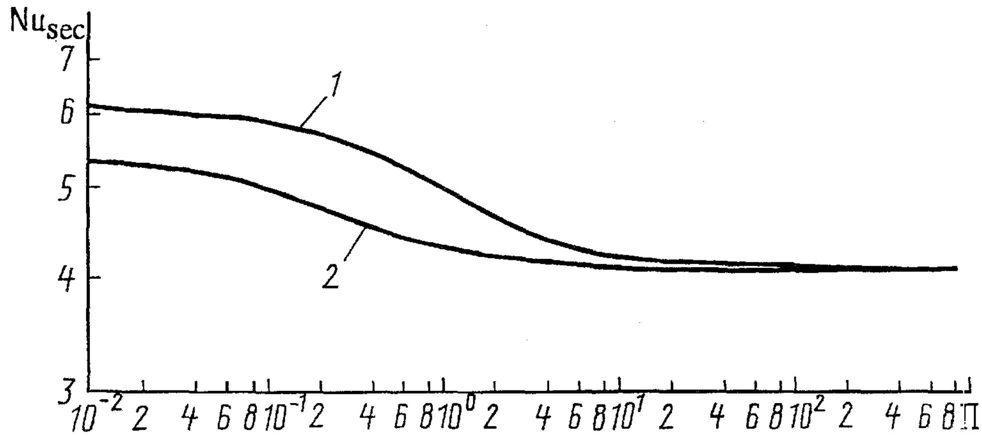

Fig. 3. Dependence of the integral mean limiting Nusselt number Nu_{sec} ($\alpha_{sec}h/h_2$) on the number Π $((\lambda_1/\lambda_2)\cdot(\delta/l)\cdot(R/l))$ for a laminar flow; the width of a source is equal to the spacing between the sources, $R/l = 0.25$: 1) Pe = 856; 2) 86.

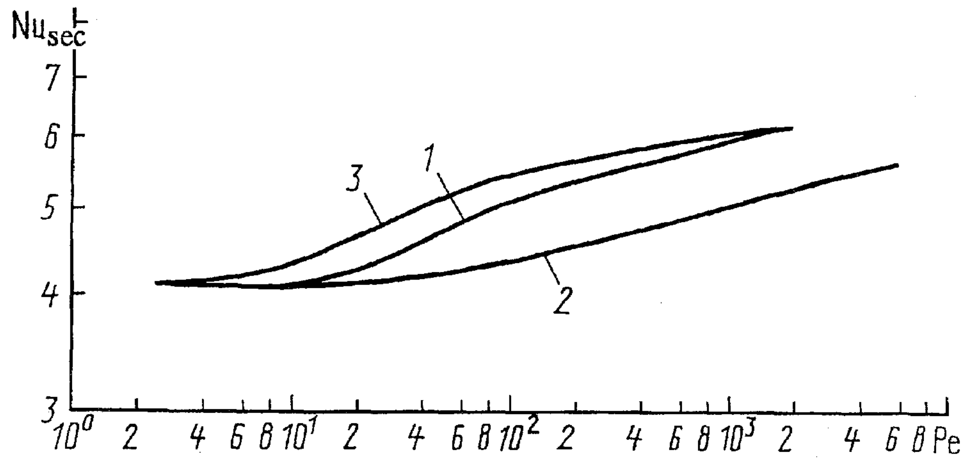

Fig. 4. Dependence of the integral mean limiting Nusselt number Nu_{sec} ($\alpha_{sec}h/\lambda_2$) for sections with heat release on the number Pe (uh/a) for a laminar flow; the width of a source is equal to the spacing between the sources, $R/l = 0.25$: 1) $\Pi^* = 0.0943$; 2) 0.943; 3) 0.00943.

Thus, the increase in the heat transfer coefficient at the site of the source in relation to the heat transfer coefficient at this site in the case of uniform heat supply over the entire length of the channel depends substantially on the thermal conductivity and thickness of the wall, and, for example, for air at $\delta/l = 0.1$, $R/l = 0.25$, and Pe = 856 already at $\lambda_1 = 5$ W/(m·K) the difference between these heat transfer coefficients amounts to no more than 8% (for the maximum heat transfer coefficient in the region of the source, no more than 13%). The dependence of the Nusselt number on the number Pe at a distance from the inlet of the channel is presented in Fig. 4.

Consequently, the number Nu_{sec} at a distance from the inlet of the channel is not a constant value, as in the case of a uniform heat supply ($Nu_\infty = 4.12$), but depends on the number Pe. The same result also follows from the solution of the nonconjugate problem of heat transfer, i.e., without allowance for the axial thermal conductivity of the wall.

Another important characteristic of heat transfer under conditions of discrete heat supply with allowance for conductive spreading of heat along the channel wall is the power removed by convection to the air in the channel directly from the surface of the source (or the convective heat flux density). In the considered case of an adiabatic outer surface of the channel wall the total power of heat release by the source P_{sec} is represented as a sum of powers: that removed by convection directly from the surface of the source $P_{conv.sec}$ and that conducted away into

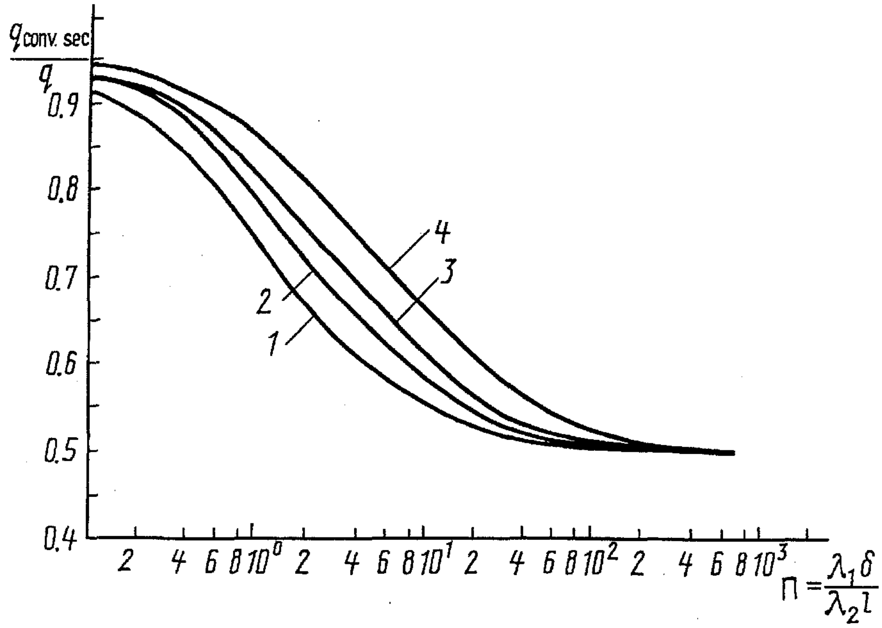

Fig. 5. Dependence of the ratio of $q_{conv.sec}$ to q on the numbers Π and Pe_h for $h/l = 0.5$ and $l/L = 0.5$: 1) $Pe_h = 143$; 2) 571; 3) 1142; 4) 4489.

the portions of the wall adjacent to the source (on the left and the right) for subsequent transfer likewise to the air in the channel already in the spacings between the sources $P_{cond.sec.}$, i.e., $P_{sec} = P_{conv.sec} + P_{cond.sec}$. For sources in the form of transverse bands (a two-dimensional problem) $P_{sec} = ql$, where q is the density of the heat flux from a source, and $P_{conv.sec} = q_{conv.sec}l$, where $q_{conv.sec}$ is the average convective heat flux density over the source length,

$$q_{conv.sec} = \lambda_2/(Rl) \int_{nL}^{(nL+l)} \left. \frac{\partial \vartheta}{\partial \xi} \right|_{\xi=1} dx .$$

Results of a calculation of the ratio $q_{conv.sec}/q$ (or, which is the same, the fraction of the power removed by convection directly from the surface of the source $P_{conv.sec}/P_{sec}$) as a function of the number Π in a wide range of the determining parameters are presented in Fig. 5.

Knowing α_{sec} and $q_{conv.sec}$, it is not difficult to determine the superheating of the wall temperature in the region of the source relative to the air (liquid) temperature under the source: $\bar{\theta}_{sec} = \bar{\vartheta}_{sec} - \vartheta_{air} = q_{conv.sec}/\alpha_{sec}$ ($Nu_{sec} = \alpha_{sec}h/\lambda_2$). Investigation of the change in the values of Nu_{sec} and $q_{conv.sec}$ as a function of the determining parameters for the subsequent determination of $\bar{\theta}_{sec}$ for solving practical problems seems to be more convenient than direct determination of superheating from a theoretical solution of the problem.

Results of calculations of the local numbers Nu_x also show that for certain ratios between Π and Pe (usually at a low thermal conductivity of the wall λ_1 and a small thickness δ) at the middle of the section without heat supply a change in the heat flux direction to the opposite is possible, i.e., the heat flux is directed from the liquid to the walls, and this corresponds to negative values of the local Nusselt numbers. This can be explained by a rate of heat transfer by convection along the flow between sources that exceeds the rate of heat conduction along the wall.

A calculation of the change in the local Nusselt numbers in the entrance section, for example, for slug flow (see Fig. 2), shows that at Pe = 856 the deviation of the maximum Nusselt number in the of heat release section from its limiting value at large values of x/h amounts to no more than 9% already in the fifth section of heat supply. In this case, starting from the second or third section the ratio of the local Nusselt number with discrete heat supply to the local Nusselt number obtained from the solution of the corresponding conjugate problem with uniform heat supply depends weakly on the location of the source, increasing somewhat with distance from the inlet of the channel.

We should also note that the limiting average Nusselt number for the heat release section increases with increase in R/l.

The method suggested in the present work seems to be efficient for solving problems in the case of an arbitrary periodic change in the heat flux density on the wall. In particular, the case where the width of a source is not equal to the spacing between the sources can be analyzed similarly.

Using the formulas presented or Figs. 3 and 4 one can determine the limiting values of the Nusselt number for the sections with heat release when conducting practical calculations of heat transfer in long plane channels with discrete sources of heat generation on the walls.

NOTATION

t_1, t_2, temperatures of the wall and liquid, respectively; t_0, flow temperature at the inlet to the channel; U_m, $\overline{U}$, maximum and mean velocities of liquid in laminar flow; U_0, liquid velocity in slug flow; q, density of heat flux from a discrete heat generation source; λ_1, λ_2, thermal conductivities of the wall material and liquid; a, thermal diffusivity of liquid; δ, channel wall thickness; $h = 2R$, distance between the walls; l, width of heat generation source, equal to the spacing between the sources; L, spacing of the sources along the heat carrier path and the coefficient in the tables; $\Pi^* = \Pi R/l$.

REFERENCES

1. É. Ya. Épik, V. D. Mel'nik and T. T. Suprun, Prom. Teplotekh., **11**, No. 1, 23-29 (1989).
2. V. B. Gidalevich, Yu. P. Mironenko, Yu. E. Spokoinyi, et al., Problems of Radioelectronics [in Russian], Issue 3 (1983).
3. M. M. Korotkevich, Methods of Calculation of a Thermal Regime and Automated Design of the Structures of Air-Cooled Apparatus, Candidate's Dissertation, St. Petersburg (1990).
4. E. P. Incropera and I. S. Kerby, Int. J. Heat Mass Transfer, **29**, No. 7, 1051-1058 (1986).
5. E. M. Sparrow, A. Garcia, and W. Chuck, Int. J. Heat Mass Transfer, **30**, No. 1, 175-185 (1987).
6. B. S. Petukhov, Heat Transfer and Resistance in Laminar Liquid Flow in Tubes [in Russian], Moscow (1967).
7. Applied Scientific Research, Section A, **8**, No. 5 (1959).
8. A. N. Zharov, Problems of Radioelectronics [in Russian], Issue 2 (1992).